AVAILABILITY ANALYSIS OF A TWO-UNIT STANDBY SYSTEM WITH DELAYED REPLACEMENT UNDER IMPERFECT SWITCHING

By

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ABSTRACT: The author has considered a complex system composed of two identical units one operative and other in cold standby. Each of the units of the system has three modes viz., normal, degraded and total failure. The system fails when both units fail totally, and may also fail due to common cause failure. The time taken in replacement of a failed unit by a standby unit is a random variable. The switching device for standby unit is not perfect.
KEYWORDS: Availability, Laplace Transform, Repairable system.

INTRODUCTION : In this chapter the author has considered a complex system composed of two identical units one operative and other in cold standby. Each of the units of the system has three modes viz., normal degraded and total failure. The system fails when both units fail totally, and may also fail due to common cause failure. The time taken in replacement of a failed unit by a standby unit is a random variable. The switching device for standby unit is not perfect. Failure rates of the units are distinct. Inspection rate, replacement rate and repair rates are distributed quite generally. The system has been analyses to determine the reliability measures viz., MTSF, steady state availability, expected profit etc., by using theory of supplementary variable technique. These equations are solved with the help of Laplace transform technique. Certain particular cases are analyses. Also, numerical illustration with various graphs have been given at the end to connect the model with physical situation.

ASSUMPTIONS :

(i) A cold standby system comprises two similar units. Each unit has three modes: normal (N), Degraded (D) and total failure (F). The units are said to be N,D and F units if they are in their respective modes.

(ii) The replacement time of a failed unit is not negligible but is a random variable called delay time.

(iii) The system fails on the total failure of its both units and also breaks down completely due to common cause failure.

(iv) In the normal state, the system is inspected with the general inspection rate distribution.

(v) Failure time distributions are negative exponential, while the repair and delay time distributions are arbitrary.

(vi) A single service facility is available to repair a D-unit, a F-unit and to activate the cold standby unit. The repair facility is not always with the system but can be made available instantaneously whenever needed.

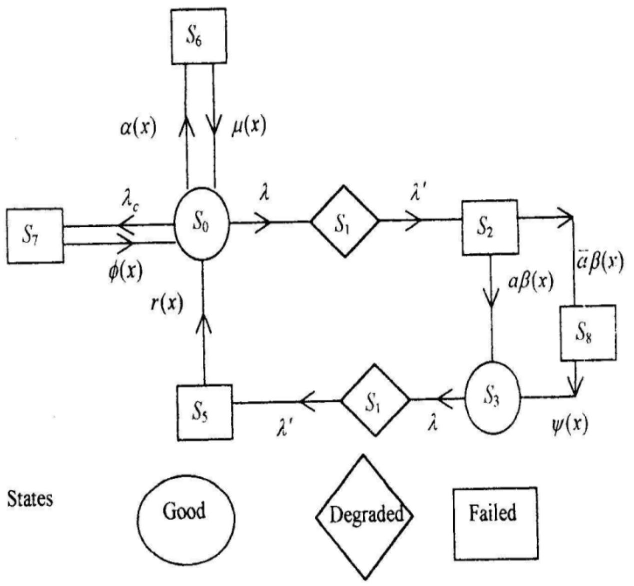
(vii) Initially system works with normal efficiency after inspection.

(viii) A repaired unit works as good as new.

(ix) Switching over device for standby unit is not perfect.

NOTATIONS:

$D / D_t / D_x$	: $\frac{d}{dt} / \frac{\partial}{\partial t} / \frac{\partial}{\partial x}$
α	: Probability of successful operations of switching over device
λ / λ'	: Constant failure rates from N to D/D to F
λ_c	: Constant common cause failure rate
$\alpha(x) / \mu(x)$	: Inspection rate from S_0 to S_6 / S_6 to S_0
$r(x) / \phi(x)$	: General repair rate from S_5 to S_0 / S_7 to S_0
$\psi(x)$	: General repair rate from S_8 to S_0
$\beta(x)$	: Switching rate from S_2 to S_3
$P_i(t)$	: Probabilities in the S_i state where i= 1, 3, 4
`$P_1(x,t)$	: Probabilities at the S_i state where i = 0, 2, 5, 6, 7, 8.



Transition diagram

FORMULATION OF MATHEMATICAL PROBLEM : By elementary probability considerations and continuity arguments, the difference-differential equations for the stochastic process, which is continuous in time and discrete in space, are

$$\left[D_x+s+\lambda+\lambda_c+\alpha(x)\right]\bar{P}_0(x,s)=0 \tag{1}$$

$$[s+\lambda']\bar{P}_1(s)=\lambda\bar{P}_0(s) \tag{2}$$

$$\left[D_x+s+\beta(x)\right]\bar{P}_2(x,s)=0 \tag{3}$$

$$[s-\lambda]\bar{P}_3(s)=\int_0^\infty \alpha\beta(x)\bar{P}_2(x,s)dx+\int_0^\infty \bar{P}_8(x,s)\psi(x)dx \tag{4}$$

$$[s+\lambda']\bar{P}_4(s)=\lambda\bar{P}_3(s) \tag{5}$$

$$\left[D_x+s+r(x)\right]\bar{P}_5(x,s)=0 \tag{6}$$

$$\left[D_x+s+\mu(x)\right]\bar{P}_6(x,s)=0 \tag{7}$$

$$\left[D_x+s+\phi(x)\right]\bar{P}_7(x,s)=0 \tag{8}$$

$$\left[D_x+s+\psi(x)\right]\bar{P}_8(x,s)=0 \tag{9}$$

Boundary Conditions :

$$P_0(0,s)=\int_0^\infty \bar{P}_5(x,s)r(x)dx+\int_0^\infty \bar{P}_7(x,s)\phi(x)dx+\int_0^\infty \bar{P}_6(x,s)\mu(x)dx \tag{10}$$

$$\bar{P}_2(0,s)=\lambda'\bar{P}_1(s) \tag{11}$$

$$\bar{P}_5(0,s)=\lambda'\bar{P}_4(s) \tag{12}$$

$$\bar{P}_6(0,s)=\int_0^\infty \bar{P}_0(x,s)\alpha(x)dx \tag{13}$$

$$\bar{P}_7(0,s)=\lambda_c\bar{P}_0(s) \tag{14}$$

$$\bar{P}_8(0,s)=\int_0^\infty \bar{P}_2(x,s)\bar{\alpha}\beta(x)dx \tag{15}$$

Initial Conditions :

$\bar{P}_0(0,)=1$ and other state probabilities are zero at $t=0$

SOLUTION OF THE MODEL : On integrating equations (1) through (9) and using (10) to (15), after minor simplification, we may obtain:

$$\bar{P}_0(s)=\frac{1}{T(s)}D_\alpha(s+\lambda+\lambda_c) \tag{16}$$

$$\bar{P}_1(s)=\frac{\lambda}{s+\lambda'}.\frac{1}{T(s)}D_\alpha(s+\lambda+\lambda_c) \tag{17}$$

$$\bar{P}_2(s)=\frac{\lambda\lambda'}{s+\lambda'}.\frac{1}{T(s)}D_\alpha(s+\lambda+\lambda_c)D_\beta(s) \tag{18}$$

$$\bar{P}_3(s)=\frac{\lambda\lambda'}{(s+\lambda)(s+\lambda')}.\frac{1}{T(s)}D_\alpha(s+\lambda+\lambda_c)\bar{S}^\beta(s)\left(\alpha+\bar{\alpha}\bar{S}^\psi(s)\right) \tag{19}$$

$$\bar{P}_4(s)=\frac{\lambda^2\lambda'}{(s+\lambda)(s+\lambda)^2}.\frac{1}{T(s)}.D_\alpha(s+\lambda+\lambda_c).\bar{S}^\beta(s)\left(\alpha+\bar{\alpha}\bar{S}^\psi(s)\right) \tag{20}$$

$$\bar{P}_5(s)=\frac{\lambda^2\lambda'}{(s+\lambda')^2(s+\lambda)}.\frac{D_r(s)}{T(s)}D_\alpha(s+\lambda+\lambda_c)\bar{S}^\beta(s)\left(\alpha+\bar{\alpha}\bar{S}^\psi(s)\right) \quad (21)$$

$$\bar{P}_6(s)=\frac{1}{T(s)}.\bar{S}^\alpha(s+\lambda+\lambda_c)D_\mu(s) \quad (22)$$

$$\bar{P}_7(s)=\lambda_c.\frac{1}{T(s)}.D_\alpha(s+\lambda+\lambda_c)D_\phi(s) \quad (23)$$

$$\bar{P}_8(s)=\frac{\bar{\alpha}\lambda\lambda'}{(s+\lambda)T(s)}.D_\alpha(s+\lambda+\lambda_c)D_\psi(s)\bar{S}^\beta(s) \quad (24)$$

Where

$$T(s)=1-\frac{\lambda^2\lambda'^2}{(s+\lambda)^2(s+\lambda)}.\bar{S}^\beta(s)\bar{S}^r(s)\left(\alpha+\bar{\alpha}\bar{S}^\psi(s)\right)-\lambda_c D_\alpha(s+\lambda+\lambda_c).\bar{S}^\phi(s)$$

$$-\bar{S}^\alpha(s+\lambda+\lambda_c).\bar{S}^\mu(s)$$

$$D_\theta(s)=\frac{1-\bar{S}^\theta(s)}{s}$$

LAPLACE TRANSFORMS OF UP AND DOWN STATE PROBABILITIES :

$$\bar{P}_{up}(s)=\frac{D_\alpha(s+\lambda+\lambda_c)}{T(s)}\left[1+\frac{\lambda}{s+\lambda'}+\frac{\lambda\lambda'}{(s+\lambda)(s+\lambda')}\bar{S}^\beta(s)\left(\alpha+\bar{\alpha}\bar{S}^\psi(s)\right)\right]$$

$$+\frac{\lambda^2\lambda'}{(s+\lambda)^2(s+\lambda)}\bar{S}^\beta(s)\left(\alpha+\bar{\alpha}\bar{S}^\psi(s)\right) \quad (25)$$

$$\bar{P}_{down}(s)=\frac{D_\alpha(s+\lambda+\lambda_c)}{T(s)}\left[\frac{\lambda\lambda'}{s+\lambda'}D_\beta(s)+\frac{\lambda^2\lambda'^2}{(s+\lambda)^2(s+\lambda)}\bar{S}^\beta(s)\left(\alpha+\bar{\alpha}\bar{S}^\psi(s)\right)\right.$$

$$\times D_r(s)+\frac{1}{T(s)}\frac{\bar{S}^\alpha(s+\lambda+\lambda_c)}{D_\alpha(s+\lambda+\lambda_c)}D_\mu(s)+\lambda_c D\phi(s)+\frac{\bar{\alpha}\lambda\lambda'}{s+\lambda'}\bar{S}^\beta(s)D_\psi(s)] \quad (26)$$

It is interesting to note that

$$\bar{P}_{up}(s)+\bar{P}_{down}(s)=\frac{1}{s} \quad (27)$$

COST ANALYSIS : The expected net profit function is given as follows:

Expected total profit in $(0,t)$ = expected total revenue in $(0,t)$ - expected total cost $(0,t)$

$$=C_1\int_0^t R(t)\,dt-C_2 t$$

where C_1 and C_2 are revenue per unit up time and repair cost per unit time respectively.

NUMERICAL COMPUTATION :

(a) Availability Analysis : Setting $\alpha=0.8$; $\lambda=0.015$; $\lambda'=0.025$; $\beta=0.75$; $\psi=r=\mu=\phi-1$; $\alpha=0.85$ in equation (36), we get

(b) MTSF Analysis : Setting $\lambda=0.001$; $\alpha=0.8$; $\lambda'=0.002$; $\alpha=0.8$ in equation (48), we get

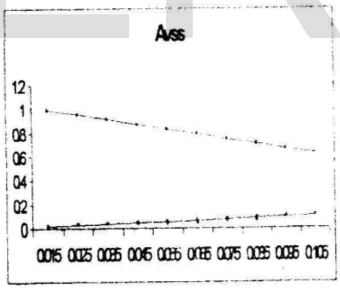


Fig. 1.

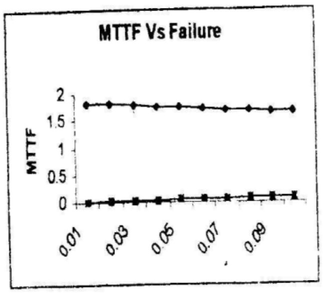


Fig. 2.

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